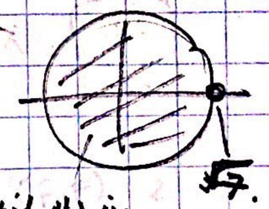
$f(x,y) = \cancel{5}\, e^{(x^2 - y^2 + 4y)}$

14.4 HW Q9 · Find abs. max/mins of $f(x,y) = x^2 + 10xy + y^2$ on the disk $D = \{\underbrace{x^2+y^2}_{g(x,y)} \le 7\}$.



(crit. pts on bdy). ① $\nabla f = \lambda \nabla g$

② (crit. pts in interior) $\nabla f = 0$.

$$\begin{bmatrix} 2x+10y \\ 2y+10x \end{bmatrix} = \lambda \begin{bmatrix} 2x \\ 2y \end{bmatrix} \rightsquigarrow \begin{array}{ll} \text{(i)} & 2x+10y = \lambda 2x \\ \text{(ii)} & 2y+10x = \lambda 2y \end{array}$$

edge of region → (iii) $x^2 + y^2 = 7$.

(i) $\lambda = \dfrac{2x+10y}{2x} = \dfrac{2x}{2x} + \dfrac{10y}{2x} = 1 + 5\dfrac{y}{x}$

(ii) $\lambda = \dfrac{2y}{2y} + \dfrac{10x}{2y} = 1 + 5\dfrac{x}{y}$.

(i) & (ii) $\Rightarrow 1 + \dfrac{5y}{x} = 1 + \dfrac{5x}{y}$

$\Rightarrow 5\dfrac{y}{x} = 5\dfrac{x}{y}$

$\Rightarrow \underline{y^2 = x^2}$

(iii) $x^2 + x^2 = 7$
$\Rightarrow 2x^2 = 7$.
$\Rightarrow x = \pm\sqrt{7/2}$

(iii) $\frac{7}{2} + y^2 = 7$.
$\Rightarrow y^2 = 7/2$
$\Rightarrow y = \pm\sqrt{7/2}$.

⚠ looks like $\left\{\left(+\sqrt{\tfrac{7}{2}}, \sqrt{\tfrac{7}{2}}\right), \left(+\sqrt{\tfrac{7}{2}}, -\sqrt{\tfrac{7}{2}}\right), \left(-\sqrt{\tfrac{7}{2}}, \sqrt{\tfrac{7}{2}}\right), \left(-\sqrt{\tfrac{7}{2}}, -\sqrt{\tfrac{7}{2}}\right)\right\}$.

Not all these may actually crit. points - but these are candidates on the boundary of the disk.

② crit. pts in the interior... $\nabla f = 0$

$$\begin{pmatrix} 2x+10y = 0 \\ 2y+10x = 0 \end{pmatrix} \rightsquigarrow x = -5y$$

$\rightsquigarrow 2y + 10(-5y) = 0$
$\Rightarrow y(-48) = 0 \quad \Rightarrow y = 0$.
$\Rightarrow x = 0$.

So the only crit. pt. is $(0,0)$.

③ Plug each pt. in to $f(x,y)$ & see which is the biggest/smallest.

Practice Quiz 4 Q3. $f(x,y) = 3xy$ constraint: $\underbrace{x+y}_{g} = 14$.

$\nabla f = \lambda \nabla g$

$\begin{bmatrix} 3y \\ 3x \end{bmatrix} = \lambda \begin{pmatrix} 1 \\ 1 \end{pmatrix} \rightsquigarrow$ (i) $3y = \lambda$, (ii) $3x = \lambda$, (iii) $x+y = 14$ $\Big\} \Rightarrow x = y$

$\Rightarrow x = y = 7$.

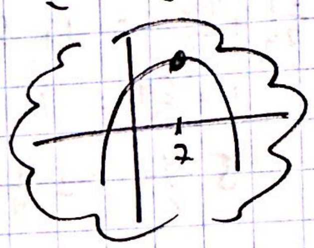
Q: Is $(7,7)$ a max/min/saddle?
We know how to do this in one-variable functions. (calc 1).

$f(x) = \ldots$
$f'(x) = 0 \rightsquigarrow$ finds crit. pts. x_0.
$f''(x_0) = \begin{cases} <0 & \text{max} \\ >0 & \text{min.} \\ =0 & \ldots \end{cases}$

Sometimes
We can turn a function of 2-variables into a function of one variable...
constraint: $y = 14 - x$.
(plug into f)

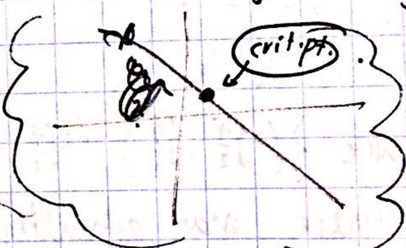
$f(x) = 3x(14-x) = 42x - 3x^2$

$14 \times 3 = 42$



We know that the crit. pt is $x_0 = 7$
Double-deriv. test...
$f'(x) = 42 - 6x$.
$f''(x) = -6$
in particular @ $x = x_0 = 7$, $f''(7) = -6 \Rightarrow 7$ is a local max.

Practice Quiz 4 Q4 : $f(x,y) = x^2 + y^2$ (constraint): $2x + 5y = 2 \rightsquigarrow y = \frac{-2x}{5} + \frac{2}{5}$.



↳ (find crit. points) ...

↳ (test what kind of crit. pt.?)

(i) ✓ do what we did above in Q3.

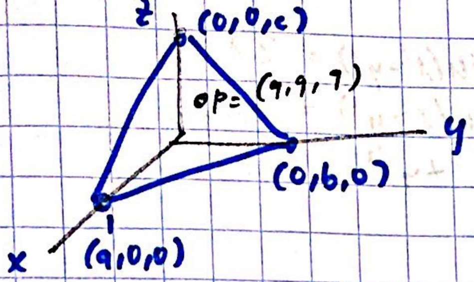
(ii) want to study how f changes as x gets "very positive" or x gets "very negative".
ie... as $x \to \infty$
$x \to -\infty$.

Since $y = \frac{-2x}{5} + 2/5$. As $x \to \infty \Rightarrow y \to -\infty$
As $x \to -\infty \Rightarrow y \to \infty$

In either case,
$f(x,y) = x^2 + y^2 \longrightarrow \infty$.
$\Rightarrow$ critical pt is a min.

14.4 HW Q8

A plane $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$ $(a,b,c>0)$



$V=\frac{1}{6}abc = f(a,b,c)$

want min.

constraint : ~~point~~ planes should contain $p=(9,9,9)$

i.e. $\underbrace{\frac{9}{a}+\frac{9}{b}+\frac{9}{c}}_{g(a,b,c)}=1$.

$\nabla f = \lambda \nabla g$

$$\begin{pmatrix}\frac{1}{6}bc\\ \frac{1}{6}ac\\ \frac{1}{6}ab\end{pmatrix}=\lambda\begin{pmatrix}-9a^{-2}\\ -9b^{-2}\\ -9c^{-2}\end{pmatrix} \rightsquigarrow \begin{array}{l}(i)\ \frac{1}{6}bc=\frac{-9}{a^2}\lambda\\ (ii)\ \frac{1}{6}ac=-9/b^2\ \lambda\\ (iii)\ \frac{1}{6}ab=-9/c^2\ \lambda\\ (iv)\ \frac{9}{a}+\frac{9}{b}+\frac{9}{c}=1\end{array} \rightsquigarrow \begin{array}{ll}\lambda=\frac{a^2bc}{-54} & (i)'\\ \lambda=\frac{ab^2c}{-54} & (ii)'\\ \lambda=abc^2/-54 & (iii)'\end{array}$$

$(i)'=(ii)' \rightsquigarrow \frac{a^2bc}{(-54)}=\frac{ab^2c}{(-54)} \rightsquigarrow a^2bc=ab^2c$

$(ii)'=(iii)' \rightsquigarrow \frac{ab^2c}{(-54)}=\frac{abc^2}{(-54)} \rightsquigarrow ab^2c=abc^2$

⚠ might want to cancel c's. $a^2b=ab^2$ (but you have to assume $c\neq 0$)

$a^2bc-ab^2c=0$

$abc(a-b)=0$

$\Rightarrow$ either $(abc=0)$ $\longrightarrow$ ✗ since we assumed $a,b,c>0$

or $(a-b=0) \Rightarrow a=b$.

$ab^2c-abc^2=0$

$abc(b-c)=0$

$\Rightarrow b-c=0 \rightarrow b=c$.

$\Rightarrow a=b=c$.

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14.4 HW Q10. $f(x,y) = 5\cos(x^2-y^2)$ (constraint): $\underbrace{x^2+y^2}_{g}=1$

$\nabla f = \lambda \nabla g$ $\qquad 5(-\sin(\dots))\cdot 2x = -10x\sin(..)$

$$\begin{pmatrix} -10x\sin(x^2-y^2) \\ +10y\sin(x^2-y^2) \end{pmatrix} = \lambda \begin{pmatrix} 2x \\ 2y \end{pmatrix} \rightsquigarrow \begin{array}{l} \text{(i) } -10x\sin(x^2-y^2) = 2\lambda x \\ \text{(ii) } 10y\sin(x^2-y^2) = 2\lambda y. \\ \text{(iii) } x^2+y^2 = 1 \end{array}$$

(i) $-10x\sin(x^2-y^2) = 2\lambda x$

Assuming $x \neq 0$, $\therefore \lambda = -5\sin(x^2+y^2)$

(ii) $10y\sin(x^2-y^2) = 2\lambda y$

Assuming $y \neq 0$, $\therefore \lambda = 5\sin(x^2+y^2)$

$2\lambda = 0$

$\Rightarrow \lambda = 0.$

$\Rightarrow$ (i) $-10x\sin(x^2-y^2) = 0$

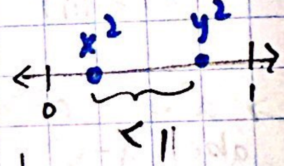
$\Rightarrow \underbrace{-10x}_{\neq 0 \text{ since } x\neq 0}\sin(x^2-y^2) = 0 \Rightarrow \sin(x^2-y^2)=0$

$\Rightarrow x^2-y^2 = \{\dots, -\pi, 0, \pi, 2\pi, \dots\}$

Since $x^2+y^2=1 \Rightarrow$ both

$0 \le |x| \le 1 \rightsquigarrow 0 \le x^2 \le 1$

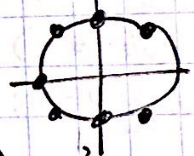
$0 \le |y| \le 1 \rightsquigarrow 0 \le y^2 \le 1$



So ... $x^2-y^2 = 0.$ $\Rightarrow x^2 = y^2$

↪ plug into constraint.

→ end up w/ $\{(\frac{1}{\sqrt2}, \frac{1}{\sqrt2}), (\frac{-1}{\sqrt2}, \frac{1}{\sqrt2}) \dots\}$.



But we also need to consider when $x=0$ or $y=0$.

$\{(0,1),(0,-1)\}$. $\qquad \{(1,0),(-1,0)\}$.