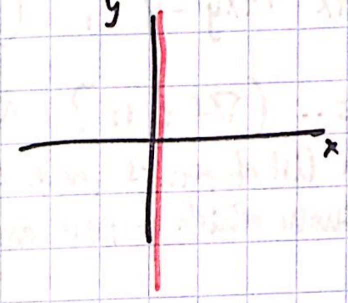
written HW

Q2: $f(x,y) = 5ye^{x} - e^{5x} - y^{5}$.

"restrict to the y-axis" ($x=0$).

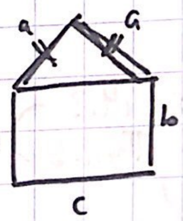


· written HW3 (1,2,3)

· 14.3 HW

· 14.3 HW.

Q3:



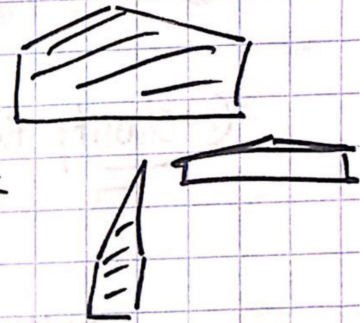
perimeter

~~length~~ (L) fixed

want to maximize area (A.)

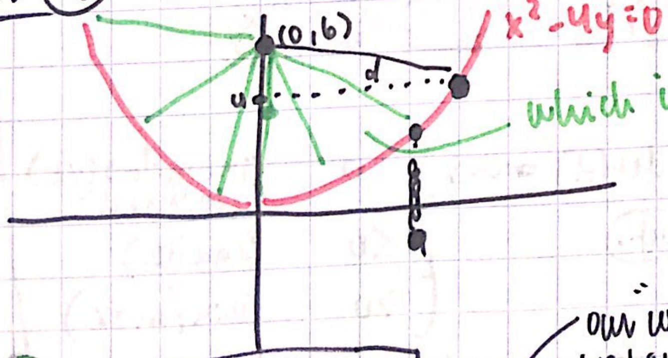
we want to optimize (maximize) a function A

w/ a constraint L.



write L, A in terms of (a, b, c)

14.3. #8) shortest distance from (0,6) to the parabola $\boxed{x^2 - 4y = 0}$.



(take deriv.)

(set = 0)

$d = \sqrt{(x-x_0)^2 + (y-y_0)^2}$

$= \sqrt{(x-0)^2 + (y-6)^2}$

the function we want to optimize (minimize)

"our constraint" is that we have to land on the parabola

ie. x & y should satisfy this

$\boxed{x = \sqrt{4y} = 2\sqrt{y}}$

$\leadsto d(y) = \sqrt{4y + (y-6)^2} = \sqrt{4y + y^2 - 12y + 36}$

$d(y) = \sqrt{y^2-8y+36}$ $\qquad d = \sqrt{(x-0)^2+(y-6)^2}$

take derivative. $d' = 0$. $\qquad d' = \frac{1}{2}\frac{1}{\sqrt{\dots}}(2y-8) = 0$. $\times 2\sqrt{\dots}$

~> solve for y. $\qquad \cancel{\times 2\sqrt{\dots}}$

~> plug into d(y).

$\Rightarrow 2y - 8 = 0$

$\Rightarrow y = 4.$

$d(y) = \sqrt{y^2-8y+36} = (y^2-8y+36)^{1/2}$

$d' = \frac{1}{2}(y^2-8y+36)^{-1/2}\cdot(2y-8) = 0$

$= \frac{1}{2(y^2-8y+36)^{1/2}}(2y-8) = 0$

$\cdot 2(y^2-8y+36)^{1/2} \qquad \cdot 2(y^2-8y+36)^{1/2}$

14.3 Q5. $f(x,y) = -4x^2+4xy-y^2$, $(D=0)$

① Find critical points.. $(\nabla f = 0)$ ~>

↳ gives you either a list of points, or a equation describing the crit. pts.

but we don't know which of these are { · maxs · mins · saddles

② Classify the crit. pts. (use Hessian matrix) (see book 14.3 Thm. 6)

$$H = \begin{bmatrix} f_{xx} & f_{yx} \\ f_{xy} & f_{yy} \end{bmatrix}$$

Thm 6: (2 variables)

(i) $\frac{\partial f}{\partial x} = 0 \quad \frac{\partial f}{\partial y} = 0$ (crit. pt.)

(ii) $D = \det(H)$ ~> $\begin{cases} =0 & \text{(inconclusive) (can't tell)} \\ <0 & \text{(saddle)} \\ >0 & \text{(min/max)} \end{cases}$

discriminant.

(iii) $\frac{\partial^2 f}{\partial x^2} = f_{xx}$ ~> $\begin{cases} >0 & \text{(min)} \\ <0 & \text{(max)} \end{cases}$

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14.3 Q6: $f(x,y,z) = -x^2 - y^2 - z^2 - xy$

① Finding where the crit. pts are – ($\nabla f = 0$)

$$\nabla f = \begin{pmatrix} -2x - y \\ -2y - x \\ -2z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \rightsquigarrow \left. \begin{aligned} -2x - y &= 0 \\ -2y - x &= 0 \\ -2z &= 0 \end{aligned} \right\}$$

system of eqns to solve for (x,y,z)

$y = -2x$

$-2(-2x) - x = 0$

$3x = 0 \Rightarrow x = 0.$

$\Rightarrow y = 0.$

$\Rightarrow z = 0.$

so our only crit. pt. is $\underline{(0,0,0)}$.

② Classify. (use Hessian matrix).

$$H = \begin{bmatrix} f_{xx} & f_{yx} & f_{zx} \\ f_{xy} & f_{yy} & f_{zy} \\ f_{xz} & f_{yz} & f_{zz} \end{bmatrix}$$

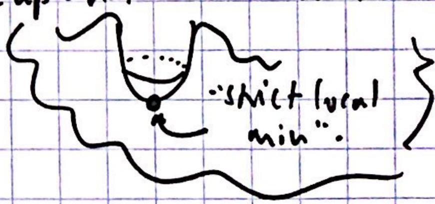
$$f_{xy} = \frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x}$$

$$= \begin{bmatrix} -2 & -1 & 0 \\ -1 & -2 & 0 \\ 0 & 0 & -2 \end{bmatrix}$$

Note: In this case, this Hessian is not in terms of x, y, z. So this classification will work for all the crit. pts.

(If you end up w/ (x,y,z)'s then you need to plug in pts before classifying).

Note: look up what "strict" means.



Want to determine if H is ...

- pos.-def. $\Rightarrow$ (local min)
- neg.-def. $\Rightarrow$ (local max)
- ~~(saddle)~~
- neither pos/neg.-def.
  - ↳ look @ det $\begin{cases} \neq 0 \Rightarrow \text{(saddle)} \\ = 0 \Rightarrow \text{("degenerate type")} \end{cases}$

How to check? "Determinant test for positive definiteness" —

Given a square matrix ..

- look @ "diagonal submatrices".
- take all their determinants.

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & & \vdots \\ \vdots & & \ddots & \vdots \\ a_{n1} & \cdots & & a_{nn} \end{bmatrix}$$

If the det's are ... $\begin{cases} \text{all} > 0, \Rightarrow H \text{ is pos.-def.} \\ \text{alternating } +/- \Rightarrow H \text{ is neg.-def.} \\ \text{all} \neq 0 \Rightarrow H \text{ is a saddle.} \end{cases}$

e.g. $\begin{bmatrix} -2 & -1 & 0 \\ -1 & -2 & 0 \\ 0 & 0 & -2 \end{bmatrix}$

det A = −2
det B = 3
det C = −6.

Ⓐ $[-2]$ Ⓑ $\begin{bmatrix} -2 & -1 \\ -1 & -2 \end{bmatrix}$

Ⓒ $\begin{bmatrix} -2 & -1 & 0 \\ -1 & -2 & 0 \\ 0 & 0 & -2 \end{bmatrix}$

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