(practice)

a 1 — 13.3.

2 — ⓘ?.

3 — 13.5

4 — 13.4

5 — 14.1

· chain rule : D (f∘g)

[redacted] "~~First special case~~" ~~is: outside function is real-valued~~.

13.4 Q5. $\frac{\partial g}{\partial \theta}$ @ $(r,\theta) = (2\sqrt{2}, \frac{\pi}{4})$ where $g(x,y) = \frac{1}{x+y^2}$

$f_1 \; x = 8r\cos(\theta)$

$f_2. \; y = 7r\sin(\theta)$

$f : \mathbb{R}^2 \longrightarrow \mathbb{R}^2$

$(r,\theta) \longmapsto (f_1(r,\theta), f_2(r,\theta))$

$\overset{\shortparallel}{8r\cos\theta} \quad \overset{\shortparallel}{7r\sin\theta}$.

[redacted]

want to think of g as a function of (r,θ)

$$\begin{tikzcd}
\mathbb{R}^2 \arrow[r, "f"] \arrow[rr, bend left, "g(r,\theta)"] & \mathbb{R}^2 \arrow[r, "g"] & \mathbb{R} \\
(r,\theta) \arrow[r, mapsto] & (f_1(r,\theta), f_2(r,\theta)) \arrow[r, mapsto] & g(f_1^{(r,\theta)}, f_2^{(r,\theta)})
\end{tikzcd}$$

$\overset{\shortparallel}{g(r,\theta)}$

$$\frac{\partial}{\partial\theta}(g \circ f)\Big|_{(2\sqrt{2}, \pi/4)} = \frac{\partial g}{\partial x}\frac{\partial f_1}{\partial \theta} + \frac{\partial g}{\partial y}\frac{\partial f_2}{\partial \theta} \longrightarrow ☆.$$

$$\overset{\shortparallel}{\frac{\partial g}{\partial x}\frac{dx}{d\theta}} \qquad \overset{\shortparallel}{\frac{\partial g}{\partial y}\frac{dy}{d\theta}}$$

$\frac{\partial g}{\partial x} = \frac{-1}{(x+y^2)^2}$

$\frac{\partial g}{\partial y} = \frac{-2y}{(x+y^2)^2}$

} in terms of x & y. not (r,θ)

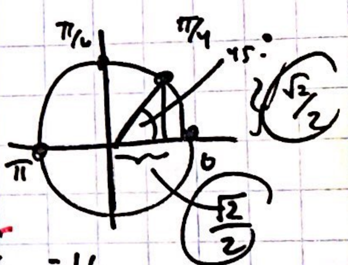
$\frac{\partial f_1}{\partial \theta} = -8r\sin\theta$

$\frac{\partial f_2}{\partial \theta} = 7r\cos\theta$

} plug in $r = 2\sqrt{2}$, $\theta = \pi/4$.

Ⓐ (i) Plug in $x = 8r\cos\theta$, $y = 7r\sin\theta$

(ii) plug in $(r,\theta) = (2\sqrt{2}, \pi/4)$



Ⓑ (i) Plug in (r,θ) into $\underset{(2\sqrt{2}, \pi/4)}{}$ $x = 8r\cos\theta$ $= 8(2\sqrt{2})\cos(\pi/4) = 16 \cdot \cancel{\sqrt{2}} \cdot \frac{\cancel{\sqrt{2}}}{\cancel{2}} = 16$

$y = 7r\sin\theta$. $= 7(2\sqrt{2})\sin(\pi/4) = 14 \cdot \cancel{\sqrt{2}} \cdot \cancel{\sqrt{2}}/\cancel{2} = 14$

(ii) plug in those into $\frac{\partial g}{\partial x}$, $\frac{\partial g}{\partial y}$.

$=$ $\frac{\partial g}{\partial x} = \frac{-1}{(x+y^2)^2} = \frac{-1}{(16+14^2)^2} = \frac{-1}{44,944}$

$\frac{\partial g}{\partial y} = \frac{-2(14)}{44944}$

plug in $(2\sqrt{2}, \pi/4)$

$\frac{\partial f_1}{\partial \theta} = -8r\sin\theta = -8(2\sqrt{2})\sqrt{2}/2 = -16$

$\frac{\partial f_2}{\partial \theta} = 7r\cos\theta = 7(2\sqrt{2})\frac{\sqrt{2}}{2} = 14$

$\frac{\partial g}{\partial x}\frac{\partial f_1}{\partial \theta} = \left(\frac{-1}{44,944} \cdot (-16)\right) = \frac{+16}{44944}$

$\frac{\partial g}{\partial y}\frac{\partial f_2}{\partial \theta} = \frac{(-2)(14)}{44,944} \cdot 14 = \frac{-392}{44944}$

} + } $= \frac{-376}{44,944}$

13.4 Q7 $f(u,v) = \langle \tan(u-1) - e^{v}, 4u^2 - 3v^2 \rangle$

$g(x,y) = \langle e^{5(x-y)}, 5(x-y) \rangle$

f,g are both $\mathbb{R}^2 \to \mathbb{R}^2$

we could do...

$$\begin{tikzcd} \mathbb{R}^2 \arrow[r, "f"] \arrow[rr, bend right] & \mathbb{R}^2 \arrow[r, "g"] & \mathbb{R}^2 \end{tikzcd}$$

or

$$\begin{tikzcd} \mathbb{R}^2 \arrow[r, "g"] \arrow[rr, bend right] & \mathbb{R}^2 \arrow[r, "f"] & \mathbb{R}^2 \end{tikzcd}$$

Want.. $\boxed{f \circ g} = f(g(\cdot\cdot))$

$f \circ g : \mathbb{R}^2$

$(x,y) \underset{g}{\longmapsto} g(x,y) \underset{f}{\longmapsto} f(g(x,y))$.

"

$f(g(x,y))$

wants to be a pair.

We have $f(u,v) = \cdots$

we've plugging in ...

$(u_{(x,y)}, v_{(x,y)}) = g(x,y)$

$= \langle e^{5(x-y)}, 5(x-y) \rangle$

$f(u,v) = \langle \tan(u-1) - e^{v}, 4u^2 - 3v^2 \rangle$

$f \circ g(x,y) = \langle \underbrace{\tan(e^{5(x-y)} - 1) - e^{5(x-y)}}_{H_1}, \underbrace{4e^{10(x-y)} - 3(5(x-y))^2}_{H_2} \rangle$

"

$H(x,y)$

gof

~~[illegible]~~ $x(u,v)$

$y(u,v)$

What we want in pt 2 DH @ (1,1)

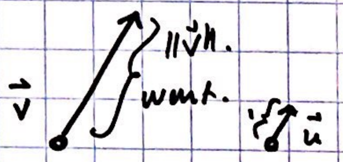
$$DH = \begin{bmatrix} \frac{\partial H_1}{\partial x} & \frac{\partial H_1}{\partial y} \\ \frac{\partial H_2}{\partial x} & \frac{\partial H_2}{\partial y} \end{bmatrix}$$

13.5 rate of change : $Df(1,1,1)(\vec{u}) = \nabla f\big|_{(1,1,1)} \cdot \vec{u}$

of some function f

@ a pt. (1,1,1)

in some direction. $\vec{u}$ – unit vector



In this problem, the direction is the normal direction to a surface.

$\underbrace{yx^2 + xy^2 + yz^2}_{g \text{ "}} = 5$.

To find normal direction to a surface..

Let the variable part ~~[illegible]~~ (left hand side) of this eqn. be a function.

$g(x,y,z) = yx^2 + xy^2 + yz^2$, then the surface is the level set $g(x,y,z) = 5$.

$\nabla g\big|_{(1,1,1)}$ is the normal vector. — This guy won't be a unit vector in general.

$\vec{v}$ "

so you will need to normalize.

$\vec{u} = \vec{v}/\|\vec{v}\|$ — ch. 11.