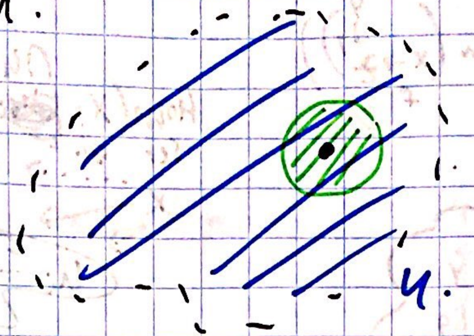
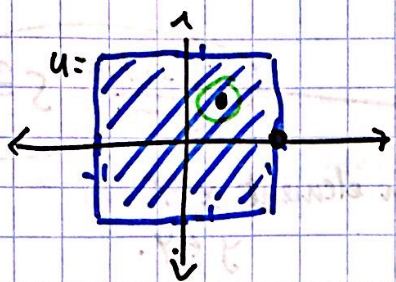
Start w/ a region U.



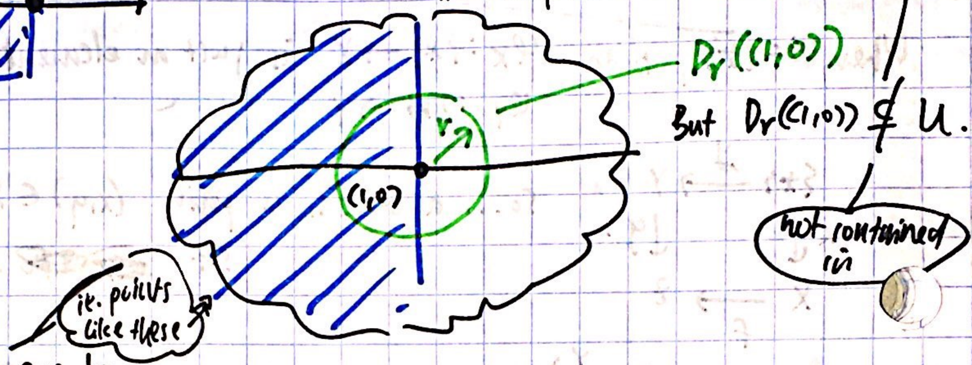
One way to show not open is if you can find a point where no matter how small of a ball you pick, it's not contained entirely within U..

eg. Take the box ... $\left\{(x,y)\in\mathbb{R}^2 : \begin{array}{l} -1\le x\le 1 \\ -1\le y\le 1\end{array}\right\}$.



Claim: This is not open.

Look at the point ... $(1,0)$

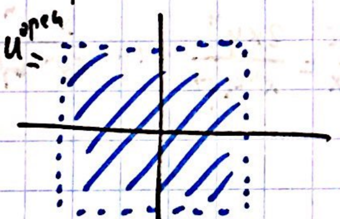


But $D_r((1,0)) \not\subseteq U$.

closed: contains its boundary points..

$\{0<x^2+y^2\le 4\}$.

eg. (of an open set) $U^{open} = \left\{(x,y)\in\mathbb{R}^2 : \begin{array}{l} -1< x< 1 \\ -1< y< 1\end{array}\right\}$.



Claim: This is open.

bounded: The whole thing can be contained inside a ball.

WH. #4) $S = \{x^2 + \frac{y^2}{9} + \frac{z^2}{4} = 1\}$.

(b) Find a point on the surface S so that its tangent plane contains the point (3,0,0).

(not the same).

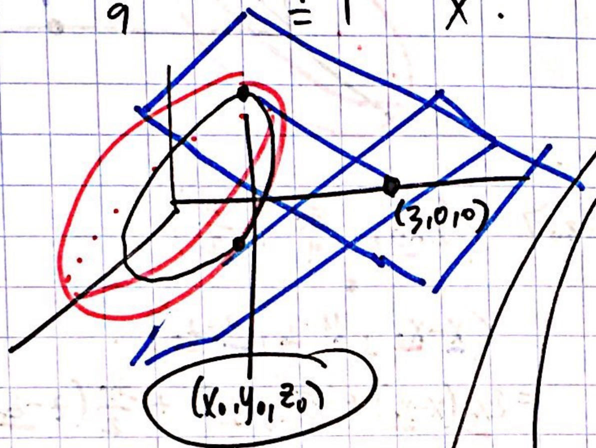
Finding a tangent plane @ the point $(3,0,0) = (x_0, y_0, z_0)$

↳ This statement only makes sense when (x_0, y_0, z_0) is in the surface S.

$(3,0,0) \notin S$ b/c if we plug in ...

$$(3)^2 + \frac{0^2}{9} + \frac{0^2}{4} \overset{?}{=} 1$$

$$9 \overset{?}{=} 1 \quad X.$$



(1) Fix some mystery pt. (x_0, y_0, z_0).

(2) Write down the eqn. of a tangent plane @ the (mystery) pt. (x_0, y_0, z_0)

(3) Since the plane should contain (3,0,0) ..

plug in $(x, y, z) = (3,0,0)$ into your eqn from (2).

$$z = \frac{\partial f}{\partial x}\Big|_{(x_0,y_0)} (x - x_0) + \frac{\partial f}{\partial y}\Big|_{(x_0,y_0)} (y - y_0) + f(x_0, y_0)$$

2 ways of writing tangent planes.

(i) $z = \frac{\partial f}{\partial x}\Big| \ldots$ — for functions $f: \mathbb{R}^2 \to \mathbb{R}$. graphs.

(ii) (for a surface..) (13.5) : $\nabla f(x_0, y_0, z_0) \cdot (x - x_0, y - y_0, z - z_0) = 0$

dot product.

When someone gives you a surface.. (i) the graph of a function $f: \mathbb{R}^2 \to \mathbb{R}$.
$(x,y) \mapsto f(x,y)$

(ii) giving a defining equation.

$\{(x,y,z)$: satisfying some equation$\}$.

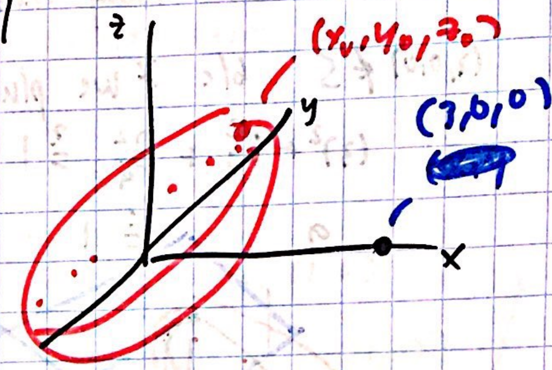
e.g. $\{(x,y,z) : x^2 + (\frac{y^2}{9}) + (\frac{z^2}{4}) = 1\}$.

Fixed (x_0, y_0, z_0) a ~~mystery~~ pt. on the surface.
Want to find a generic eqn. of a tangent plane @ (x_0, y_0, z_0).

$$S = \{ \underbrace{x^2 + \frac{y^2}{9} + \frac{z^2}{4}} = 1 \}.$$
$$= \{ f(x,y,z) = 1 \}. \quad \text{where } \left(f = x^2 + \frac{y^2}{9} + \frac{z^2}{4} \right).$$

In this case, we learned that we can describe tang. planes as...

$$\boxed{\nabla f(x_0, y_0, z_0) \underset{\text{dot}}{\cdot} \langle x - x_0, y - y_0, z - z_0 \rangle = 0.}$$



$$\nabla f = \left\langle \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z} \right\rangle.$$
$$= \left\langle 2x, \frac{2y}{9}, \underset{\frac{z}{2}}{\frac{2z}{4}} \right\rangle.$$

$$\nabla f(x_0, y_0, z_0) = \left\langle 2x_0, \frac{2y_0}{9}, \frac{z_0}{2} \right\rangle.$$

$$0 = \nabla f(x_0, y_0, z_0) \cdot \langle x - x_0, y - y_0, z - z_0 \rangle = \left\langle 2x_0, \frac{2y_0}{9}, \frac{z_0}{2} \right\rangle \cdot \langle x - x_0, y - y_0, z - z_0 \rangle.$$
$$= 2x_0(x - x_0) + \frac{2y_0}{9}(y - y_0) + \frac{z_0}{2}(z - z_0)$$

$$0 = \left(2x_0 x - 2x_0^2\right) + \left(\frac{2y_0}{9} y - \frac{2y_0^2}{9}\right) + \left(\frac{z_0}{2} z - \frac{z_0^2}{2}\right)$$
$$0 = \left(2x_0 x - \frac{2y_0}{9} y + \frac{z_0}{2} z\right) + \left(-2x_0^2 - \frac{2y_0^2}{9} - \frac{z_0^2}{2}\right).$$

We want $(3, 0, 0)$ to satisfy this ... so ...

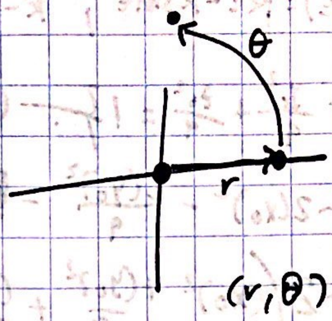
$$0 = \left(2x_0 \cdot 3 - \cancel{\frac{2y_0}{9} \cdot 0} + \cancel{\frac{z_0}{2} \cdot 0}\right) + \left(-2x_0^2 - \frac{2y_0^2}{9} - \frac{z_0^2}{2}\right)$$

$$\boxed{0 = [redacted] \; 6x_0 - 2(x_0)^2 - \frac{2(y_0)^2}{9} - \frac{(z_0)^2}{2}}$$

defining eqn of all points w/ tangent plane going through $(3, 0, 0)$. (x_0, y_0, z_0) on the surface.

(1) to make life easy, pick $z_0 := 0$. $(y_0 = 0)$
(2) use this eqn along w/ the defining eqn. of the surf.
so should be able to solve for x_0, y_0. ☺

WH #1 $f(x,y) = \begin{cases} \dfrac{2xy}{x^2+y^2} & x,y \neq 0 \\ (0,0) & x=y=0. \end{cases}$



Using polar coord's, describe level curves...

(i) (ii)

(r, θ)

"radius" "angle"

(i) $(x,y) \overset{?}{\rightsquigarrow} (r,\theta)$

(check... 11.5) $\left.\begin{array}{l} x = r\cos\theta \\ y = r\sin\theta. \end{array}\right\} \Rightarrow f(r,\theta) = \dfrac{2(r\cos\theta)(r\sin\theta)}{r^2\cos^2\theta + r^2\sin^2\theta}$

$$= \frac{2r^2\cos\theta\sin\theta}{r^2\cancel{(\cos^2\theta+\sin^2\theta)}}$$

pythag. ($\cos^2\theta + \sin^2\theta = 1$)

$$= \frac{2\cancel{r^2}\cos\theta\sin\theta}{\cancel{r^2}}$$

$f(r,\theta) = \begin{cases} 2\cos\theta\sin\theta. & r \neq 0, \theta \text{ anything}. \\ (0,0). & r=0, \theta \text{ anything}. \end{cases}$

(ii) Level curves: Given a function (real-valued) $f: \mathbb{R}^2 \to \mathbb{R}$

level curves are the sets $\{(x,y) \in \mathbb{R}^2 : f(x,y) = c\}$ (constant)

$= \{(r,\theta) \in \mathbb{R}^2 : f(r,\theta) = c\}$.

We want to describe level curves for arbitrary constants...

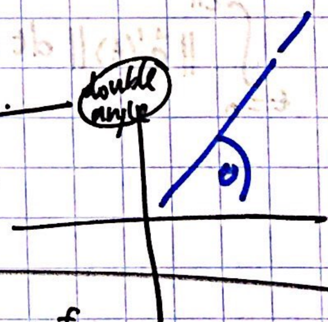
$\{(r,\theta) : f(r,\theta) = (\text{constant})\}$.

$\{(r,\theta) : \underbrace{2\cos\theta\sin\theta}_{\sin(2\theta)} = (\text{constant})\}$. double angle

$\{(r,\theta) : \sin(2\theta) = (\text{const})\}$.

ie... $2\theta = \sin^{-1}(c)$

$\theta = \frac{1}{2}\sin^{-1}(c)$



13.4 #1 $f(x,y) = 9e^{xy}$, $\vec{c}(t) = \langle \overset{c_1}{6t^2}, \overset{c_2}{t^7} \rangle$.

$\mathbb{R} \xrightarrow{c} \mathbb{R}^2 \xrightarrow{f} \mathbb{R}$

Want: $(f \circ \vec{c})'(t)$ (use chain rule..) ?

$f \circ c\,(t)$ a function of t.

$\frac{d}{dt}(f \circ c)$ should be a function of t

$$\frac{d}{dt}(f \circ c) = \frac{\partial f}{\partial x}\frac{dc_1}{dt} + \frac{\partial f}{\partial y}\frac{dc_2}{dt}$$

$\begin{cases} \dfrac{\partial f}{\partial x} = 9ye^{xy} \rightsquigarrow \text{plug in } x = c_1(t),\ y = c_2(t) \\ \dfrac{dc_1}{dt} = 12t \end{cases}$

$= 9(t^7)\, e^{(6t^2)(t^7)}$

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WH #2 $c(t) = \{Rt - R\sin t, \ R - R\cos t\}$.

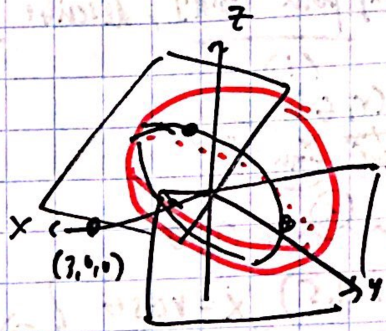
$$S = \{x_0^2 + \frac{y_0^2}{9} + \frac{z_0^2}{4} = 1\} \quad (1)$$

$$0 = 6x_0 - 2(x_0)^2 - \frac{2(y_0)^2}{9} - \frac{(z_0)^2}{} \quad (2)$$

$$0 = 6x_0 - 2\underbrace{\left((x_0)^2 + \frac{(y_0)^2}{9} + \frac{(z_0)^2}{4}\right)}_{"1}$$

$$= 6x_0 - 2$$

$$\Rightarrow \boxed{x_0 = 1/3}$$



Sanity check. This tells us that supposedly any point w/ tangent plane pas'g through (3,0,0) has x-coordinate = 1/3.

So really this boils down to points · on the surface (satisfying eqn 1)
· whose x-coord. is 1/3..

Putting these together... $x_0 + \frac{y_0^2}{9} + \frac{z_0^2}{4} = 1$

plug in 1/3.

$$\frac{1}{3} + \frac{y_0^2}{9} + \frac{z_0^2}{4} = 1$$ — defines an ellipse.

$$\frac{y_0^2}{9} + \frac{z_0^2}{4} = 2/3.$$

$c(t) = \langle Rt - R\sin t, \ R - R\cos t \rangle$.

$$\int_{t=0}^{2\pi} \|\vec{c}\,'(t)\|\, dt = \ldots$$

(1) find $\vec{c}\,'(t)$

(2) find $\| - \|$

(3) integrate.

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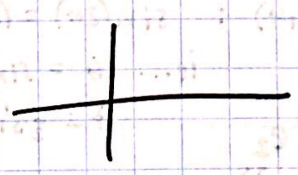
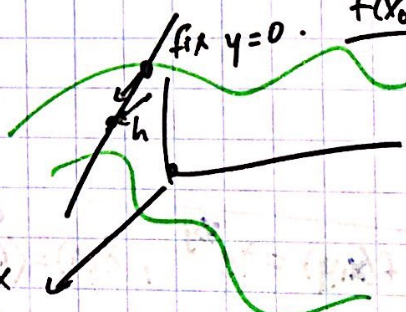


$= 8R$.

fix $y = 0$.

"write" $\frac{f(x_0+h, 0) - f(x_0, 0)}{\underbrace{(x_0+h) - x_0}_{h}}$



$$f(x,y) = \begin{cases} \frac{x^2y^4}{(x^4+6y^8)} & (x,y) \neq (0,0) \\ 0 & (x,y) = (0,0). \end{cases}$$

$\frac{\partial f}{\partial x}$ → try taking partial. (evaluating ends up w/ dividing by 0)

→ $\lim_{h \to 0} \frac{f(x_0+h, 0) - f(x_0, 0)}{h}$.

Approach along some line... like. $y = x$

$f(x,y) = \frac{x^6}{x^4+6x^8} \rightsquigarrow_{\lim_{x\to 0}} \neq 0$.

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Owen Shi
Chi
Lizzy.