The difference between a function & its graph.

e.g. $f: \mathbb{R}^1 \longrightarrow \mathbb{R}^1$ (dim in)

(dim out).

$x \longmapsto x^2 = f(x)$

domain . range

$f: \mathbb{R}^n \longrightarrow \mathbb{R}^m$

$(x_1, x_2, \dots, x_n) \mapsto (f_1(x_1 \dots x_n), f_2(x_1 \dots x_n) \dots f_m(x_1 \dots x_n))$

"vector-valued".

"scalar-valued" $f: \mathbb{R}^n \longrightarrow \mathbb{R}$

$(x_1, \dots, x_n) \longmapsto f(x_1, \dots, x_n)$

~~What is the graph~~?

e.g. (vector-valued):

$f: \mathbb{R}^3 \to \mathbb{R}^2$

$\langle x, y, z \rangle \mapsto \langle \underbrace{xy}_{f_1(x,y,z)}, \underbrace{2x + z^3}_{f_2(x,y,z)} \rangle$

(scalar-valued):

$g: \mathbb{R}^2 \to \mathbb{R}^1$

$\langle x, y \rangle \mapsto \sin(x)\cos(y)$.

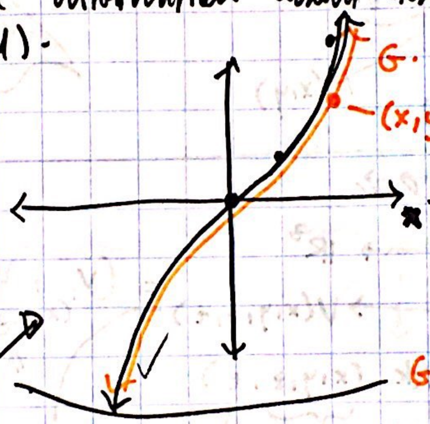
The graph of a function is a "pictorial representation" of a function — This should contain information about both its domain & range.

e.g. $f: \mathbb{R}^1 \to \mathbb{R}^1$. (scalar-valued).

$x \mapsto x^3$

domain . range .

function .



$(x, y) = (x, x^3) = (x, f(x))$

$G = \{(x, x^3)\} \subseteq \mathbb{R}^2 = \mathbb{R}^1 \times \mathbb{R}^1$

domain range

~~f: IR → IR~~ We can even interpret G (the graph) as its own function

$G: \mathbb{R}^1 \longrightarrow \mathbb{R}^2$

$x \longmapsto \langle x, x^3 \rangle$

$f: D \to \mathbb{R}^1$, $D \subseteq \mathbb{R}^2$

$(x, y) \mapsto x^2 y$

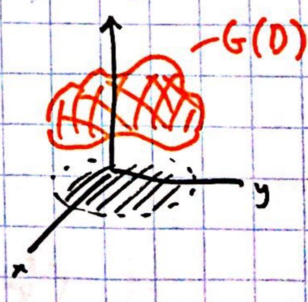
$\rightsquigarrow G \subseteq \mathbb{R}^{m+n} = \mathbb{R}^m \times \mathbb{R}^n = \mathbb{R}^2 \times \mathbb{R}^1 = \mathbb{R}^3$

dom range

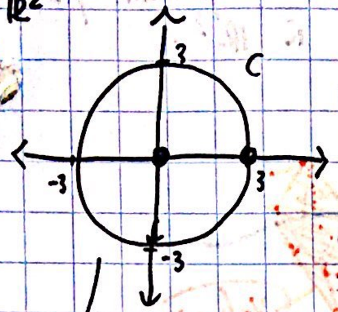
$G: D \longrightarrow \mathbb{R}^3$

same as domain(f)

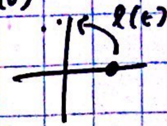
$(x, y) \longmapsto \langle x, y, f(x, y) \rangle$



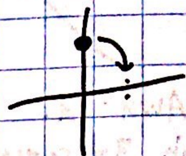
Let C be the circle w/ radius 3 & centered at the origin.
$\cap$ $\mathbb{R}^2$



Q: Find a paramatrization of C $\ell(t)$ · start at $(3,0) = \ell(0)$ · go counterclockwise



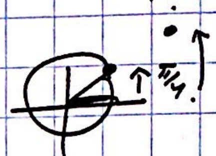
Q: Find a paramatrization · starting at $(0,3)$ · going clockwise



$x^2+y^2 = 9$

$$\ell : \mathbb{R}^1 \longrightarrow \mathbb{R}^2$$
$$t \longmapsto (\ell_1(t), \ell_2(t))$$

Any circle can be parametrized by $(\cos(t), \sin(t))$
$\mathbb{R}$ $\mathbb{R}$
radius.

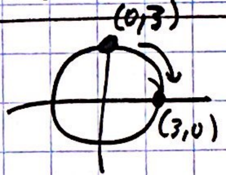


$$\ell : \mathbb{R} \longrightarrow \mathbb{R}^2$$
$$t \longmapsto \langle 3\cos(t), 3\sin(t)\rangle.$$
$$0 \longmapsto \langle 3\cos(0), 3\sin(0)\rangle$$
$$\langle 3 \;,\; 0 \rangle$$ ~ starting @ $(3,0)$

as t increases...

$$\tfrac{\pi}{4} \longmapsto \langle 3\cos(\tfrac{\pi}{4}), 3\sin(\tfrac{\pi}{4})\rangle.$$ ~ going counterclockwise.

(starting @ $(0,3)$ & going clockwise?)
$\ell(0) = (0,3)$. $\ell(\frac{\pi}{2}) = (3,0)$



$$\ell(t) \overset{?}{=} \langle 3\sin(t), 3\cos(t)\rangle$$
$$\ell(0) = \langle 3\sin(0), 3\cos(0)\rangle$$
$$= \langle 0 \;,\; 3 \rangle.$$
$$\ell(\tfrac{\pi}{2}) = \langle 3\sin(\tfrac{\pi}{2}), 3\cos(\tfrac{\pi}{2})\rangle$$
$$= \langle 3 \;,\; 0 \rangle.$$

$$\ell(t) = \langle 3\cos(\tfrac{\pi}{2} - t), 3\sin(\tfrac{\pi}{2} - t)\rangle.$$

Q: What's the difference btwn describing C as ... $\{(x,y) : x^2+y^2 = 9\}$ (1)
vs. ... $\ell(t) = \langle 3\cos(t), 3\sin(t)\rangle$. (2)

(1) describing the points.. (points (x,y) in $\mathbb{R}^2$ satisfying a relation).
(2) the paramatrization: describing C as a curve... as parametrized by a single variable t... by a path.
