BG top'l space.

classifying space of $\mathbb{Z}$.

$B\mathbb{Z} = S^1$.

Say M is a stable model cat. — pointed
— Σ, Ω form equivalences $hoM \xrightarrow{\sim} hoM$.

$h_j : Mat \longrightarrow Sets$.

$n \longmapsto h_j(n) = \mathrm{Hom}_{Mat}(j,n)$ -or- $\mathrm{Hom}(j,n)$ $\qquad n \rightrightarrows m$

$= \{n \times j \text{ matrices}\}$. $\qquad$ matrices

Yoneda: For any functor $Mat \xrightarrow{h_j} Sets$. ~~there exists an object $m \in Mat$~~ $\forall\, m \in Mat$ ~~& ...~~ there's an iso..

$h^j\ f$
$=$
$\mathrm{Hom}(j,-)$

$$Nat(mat(m,-), h_j) \cong h_j(m) = \mathrm{Hom}(j,m)$$

$\{$naturally defined column operations$\} = \{m \times j \text{ matrices}\}$.

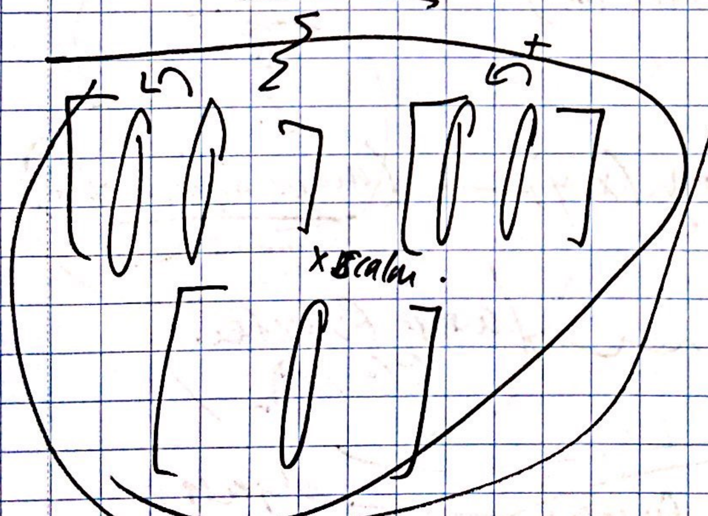
~~$h_j = \mathrm{Hom}(-,j)$~~

$\mathcal{C},\ x \in ob(\mathcal{C})$

$h_x = \mathrm{Hom}_{\mathcal{C}}(-,x)$

$h^x = \mathrm{Hom}_{\mathcal{C}}(x,-)$

$\mathcal{C} \to Set$
$\mathcal{C}^{op} \to Set$.



Yoneda embedding: "an object det. its maps out of, or in."

special case where F is of the form $\mathrm{Hom}(-,x)$ or $\mathrm{Hom}(x,-)$

$Set^{\mathcal{C}^{op}} = \mathrm{Fun}(\mathcal{C}^{op}, Set)$.

$y = よ : C \longrightarrow Fun(C^{op}, Set)$

"yo" $\quad x \longmapsto Hom_C(-, x)$

Yoneda lemma: hard to doubt

$Fun(C^{op}, Set)$

$\forall F \quad \forall c \in C$

a bij. $Fc \cong Nat(C(-,c), F)$

Yoneda Embedding is fully-faithful.

surj. $\quad$ inj.

i.e. bijections on hom-sets.

$Hom_C(x,y) \cong Hom_{Fun(C^{op}, Set)}(C(-,x), C(-,y))$

$\quad = Nat(C(-,x), \underbrace{C(-,y)}_{\text{"}F\text{"}})$ — det. objects

$\quad = C(-,y)$ of x

$\quad = C(x,y)$

"an object in C is det. by its maps ~~x~~ out/in"..

ex//. [redacted] How do representing functors determine an object.

$x \in C$

$Hom_C(x,y)$ — (maps in/out of x)

(Any object $x \in C$) $\longleftrightarrow$ (a rep functor $C(-,x)$)

objects.

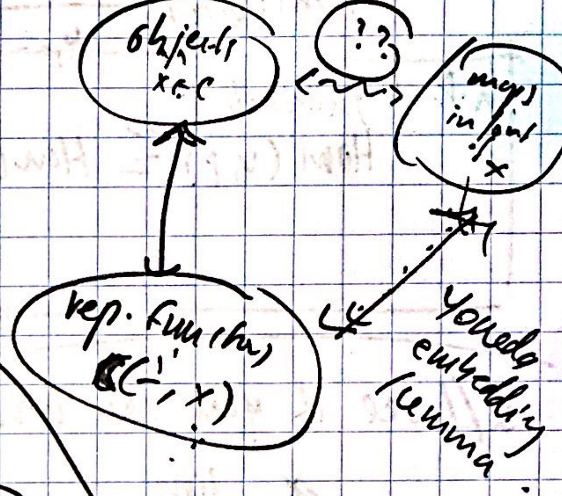
ex// "Nat'l transf's between rep'ble functors correspond to morphisms btwn the representing objects".

slogan:

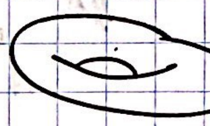
In categories the important thing is not objects but morphisms. $f : X \to Y$.

$$X \xrightarrow{f} Y$$

When we want to talk about objects, we want to turn this into some statement about morphisms.



ex// Any object $X \in \mathcal{C}$ def. a rep. functor $\mathcal{C}(-, X)$ & any rep. functor $F : \mathcal{C}^{op} \to Set$ (i.e. there's an iso natl. $F \cong \mathcal{C}(-, X)$ for some X.) corresponds to some object.



ex// Make all this precise w/ the ex. mat.

ex//. Write down U.P. of coproduct.

$$\mathrm{Hom}_{\mathcal{C}}\left(\coprod_i X_i, W\right) \cong ???$$

P = pullback $\begin{pmatrix} & B \\ & \downarrow \\ A \to & C \end{pmatrix}$ = $\lim \begin{pmatrix} & B \\ & \downarrow \\ A \to & C \end{pmatrix}$

$$\begin{tikzcd} V \arrow[r, dotted] \arrow[dr, bend right] & P \arrow[r] \arrow[d] & B \arrow[d, "\varphi_2"] \\ & A \arrow[r, "\varphi_1"'] & C \end{tikzcd} \qquad \begin{tikzcd} V \arrow[r, "g"] \arrow[d, "f"'] \arrow[dr] & B \arrow[d, "\varphi_2"] \\ A \arrow[r, "\varphi_1"'] & C \end{tikzcd}$$

$$\mathrm{Hom}(V, P) \cong \mathrm{Hom}(V, A) \times \mathrm{Hom}(V, B)$$

$$f \qquad g.$$

s.t. $\varphi_1 \circ f = \varphi_2 \circ g \in \mathrm{Hom}(V, C)$

u.p. of $\prod$:

$\mathrm{Hom}(W, \prod_i X_i) \cong \prod_i \mathrm{Hom}(W, X_i)$

Similarly. of a pullback (also a limit)

$$\begin{tikzcd}
\mathrm{Hom}(V,P) \arrow[r, "p_2\circ -"] \arrow[d, "p_1\circ -"'] \arrow[dr, phantom, "\lrcorner"] & \mathrm{Hom}(V,B) \arrow[d, "\varphi_2\circ -"] \\
\mathrm{Hom}(V,A) \arrow[r, "\varphi_1\circ -"'] & \mathrm{Hom}(V,C)
\end{tikzcd}$$

$$\begin{tikzcd}
V \arrow[r] \arrow[rr, bend left, "g"] \arrow[dr, bend right, "f"'] & P \arrow[r, "p_2"] \arrow[d, "p_1"] & B \arrow[d] \\
& A \arrow[r, "\varphi_1"'] & C
\end{tikzcd}$$

u.p. of pullback P

$$\mathrm{Hom}(V,P) \cong \mathrm{Hom}(V,A) \times_{\mathrm{Hom}(V,C)} \mathrm{Hom}(V,B)$$

pullback.

$$\begin{tikzcd}
A \times_C B \arrow[r] \arrow[d] & B \arrow[d] \\
A \arrow[r] & C
\end{tikzcd}$$

ex// see if you can write u.p.

$$\begin{tikzcd}
& \lim \arrow[dl] \arrow[dr] & \\
A \arrow[rr, "u"'] & & B
\end{tikzcd}$$

in terms of homs like

pushout

$$\begin{tikzcd}
S^1 \arrow[r] \arrow[d] & D^2 \arrow[d] \\
D^2 \arrow[r] & D^2 \cup_{S^1} D^2
\end{tikzcd}$$

$$\begin{tikzcd}
\lim \arrow[r] \arrow[d] & B \arrow[d, "id"] \\
A \arrow[r] & B
\end{tikzcd}$$