$$\begin{tikzcd} W \arrow[r, dotted, "\exists!"] \arrow[dr] \arrow[drr, bend left] & X \times Y \arrow[d] \arrow[dr] & \\ & X & Y \end{tikzcd}$$

Limits/colimits don't always exist.

$X \xrightarrow{f} Y$

$$\begin{tikzcd} W \arrow[r, dotted] \arrow[dr, bend right] & X \times Y \arrow[d, "pr_1"] \arrow[dr, "pr_2"] & \\ & X & X \end{tikzcd}$$

(initial / trivial / roles) $\Leftarrow$ u.p.: For any W w/ $W \overset{f}{\to} X$, $W \underset{g}{\to} Y$, $\exists!\ u \to X \times Y$

$W = X \times Y \times Z$

$$\begin{tikzcd} & W \arrow[d, dotted, "\exists!"] \arrow[dl, bend right] \arrow[dr, bend left] \arrow[drr, bend left] & & \\ & X \times Y \arrow[dl] \arrow[d] & & \\ X & Y & & Z \end{tikzcd}$$

"a statement of hom-sets"...

$$\mathrm{Hom}_{Set}(W, X) \times \mathrm{Hom}_{Set}(W, Y) \cong \mathrm{Hom}_{Set}(W, X \times Y)$$

$(f, g) \mapsto \exists!$

Coproducts..

u.p. $X \sqcup Y = \mathrm{colim}(X \quad Y)$ in Set.

$$\begin{tikzcd} X \arrow[d, "inc"] \arrow[drr, bend left] & Y \arrow[dl, "inc"] \arrow[dr] & \\ X \sqcup Y \arrow[rr, dotted] & & W \end{tikzcd}$$

"Given W w/ a pair of maps

$X \to W$

$Y \to W$

$\exists!\ X \sqcup Y \xrightarrow{\exists!} W$."

$\Updownarrow$

"in terms of hom-sets".

$$\mathrm{Hom}_{Set}(X, W) \times \mathrm{Hom}_{Set}(Y, W) \cong \mathrm{Hom}_{Set}(X \sqcup Y, W)$$

ex// State u.p. of an infinite product in Sets.

Say you have a bunch of sets $\{X_i\}_{i \in \mathbb{Z}}$

$W \cdots\to \prod_{i \in \mathbb{Z}} X_i = \{(x_1, \dots, x_n, \dots)\}$.

$$\begin{tikzcd} W \arrow[d] \arrow[dr] \arrow[drr] & \prod X_i \arrow[dl] \arrow[d] \arrow[dr] & \\ X_1 & X_2 & X_1 \end{tikzcd}$$

u.p. $\mathrm{Hom}_{Set}(W, \prod_i X_i) \cong$??.

Products $\leftrightarrow$ limit.

discrete diagram (i.e. no maps) — any diagram.

Yoneda Lemma

$\mathcal{C}$, Set - categories.

$$\left\{ \begin{tikzcd} \mathcal{C} \arrow[r, bend left, "F"] \arrow[r, bend right, "\mathrm{Hom}_{\mathcal{C}}(X, -)"'] \arrow[r, phantom, "\Downarrow"] & Set \end{tikzcd} \right\} \longleftrightarrow FX.$$

~~$\mathcal{C} \longrightarrow \mathrm{Fun}(\mathcal{C}^{op}, \mathrm{Set})$~~

よ = "yo" in Japanese.

$$\begin{array}{ccc}
\mathcal{C} & \longrightarrow & \mathrm{Fun}(\mathcal{C}^{op}, \mathrm{Set}) \\
 & & \Downarrow \\
x & \longmapsto & \mathcal{C}(-,x) \\
f \downarrow & & \Downarrow \alpha \\
y & \longmapsto & \mathcal{C}(-,y)
\end{array}$$

$\mathcal{C}(-,x) : \mathcal{C}^{op} \longrightarrow \mathrm{Set}$

a bunch of morphisms

$$\begin{array}{ccc}
\{\alpha_c : \mathcal{C}(c,x) & \xrightarrow[f\circ -]{\alpha_c} & \mathcal{C}(c,y)\} \\
\Uparrow & & \Uparrow \\
\mathcal{C}(c',x) & \xrightarrow[\alpha_{c'}]{} & \mathcal{C}(c',y)
\end{array}$$

$\mathcal{C} \overset{F}{\underset{G}{\Downarrow\alpha}} \mathcal{D}$ $\{\alpha_c : Fc \to Gc\}$

$$\begin{array}{ccc}
Fc & \xrightarrow{\alpha_c} & Gc \\
\downarrow & & \downarrow \\
Fc' & \xrightarrow{\alpha_{c'}} & Gc'
\end{array}$$

Yoneda embedding states

this $\text{よ} : \mathcal{C} \longrightarrow \mathrm{Fun}(\mathcal{C}^{op}, \mathrm{Set})$

is fully-faithful.

"an object is det. by maps in/out of it."

$c \xrightarrow{\varphi} x$, $x \xrightarrow{f} y$, $c \xrightarrow{f\circ\varphi ?} y$

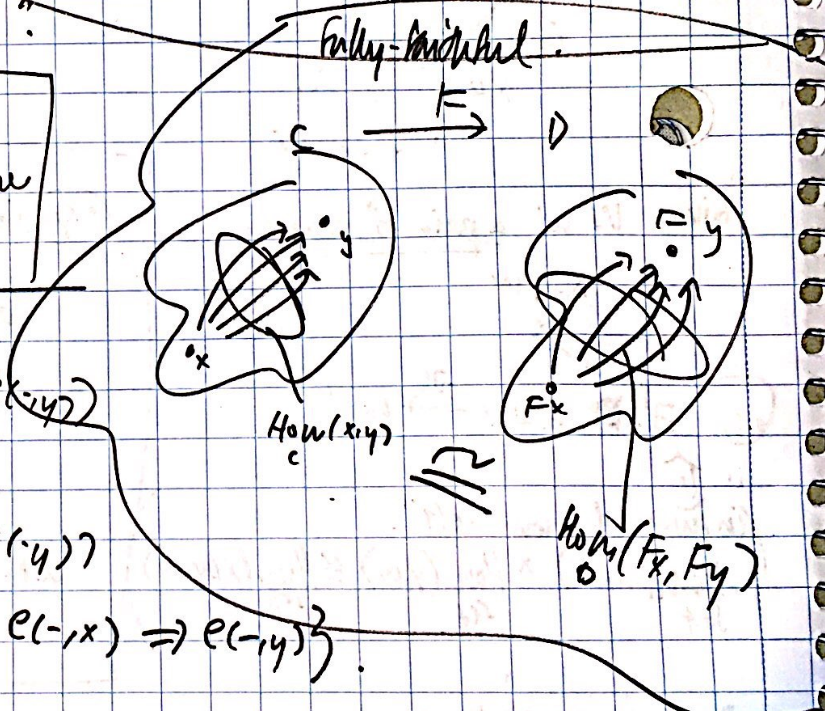
$$\begin{array}{ccc}
\mathcal{C}(c,x) & \xrightarrow[f\circ -]{\alpha_c} & \mathcal{C}(c,y) \\
(c \xrightarrow{\varphi} x) & \longmapsto & c \xrightarrow{f\circ\varphi} y
\end{array}$$

Ex. Write down the actual bijections in the case where the functor is $\text{よ} : \mathcal{C} \to \mathrm{Fun}(\mathcal{C}^{op}, \mathrm{Set})$.

$$\mathrm{Hom}_{\mathcal{C}}(x,y) \cong \mathrm{Hom}_{\mathrm{Fun}(\mathcal{C}^{op},\mathrm{Set})}(\mathcal{C}(-,x), \mathcal{C}(-,y))$$

$$\overset{\text{n}}{=} \mathrm{Nat}(\mathcal{C}(-,x), \mathcal{C}(-,y))$$

$$= \{\text{nat'l transf. } \mathcal{C}(-,x) \Rightarrow \mathcal{C}(-,y)\}.$$



~~$\text{よ} : \mathcal{C} \longrightarrow \mathrm{Fun}(\mathcal{C}^{op}, \mathrm{Set})$~~

$\mathcal{C}(-,x)$ is the functor represented by x.

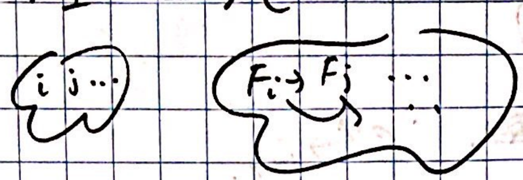
Def. $F : \mathcal{C}^{op} \to \mathrm{Set}$ is rep. by x if it's iso.

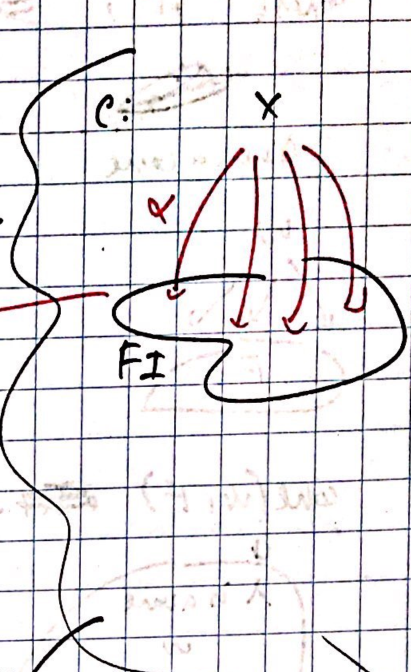
$\underset{\mathrm{iso}}{F}$

$\mathcal{C}(-,x)$

Scanned with CamScanner

$F: I \longrightarrow \mathcal{C}$





cones$(-,F): \mathcal{C}^{op} \longrightarrow Set$.

$x \longmapsto$ cones(x, F)

$\downarrow \qquad \qquad \uparrow$

$y \qquad$ cones(y,F)

$\Downarrow$ α a cone over F w/ apex x.

In particular... take $\underline{\lim F}$ (if it exists).

$\in$ ob$(\mathcal{C})$

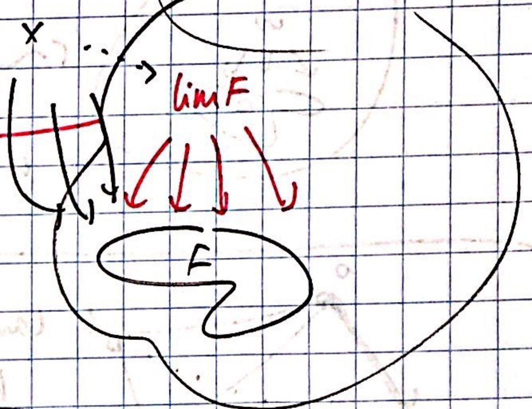
st $\lambda \in$ cones$(\lim F, F)$

$\|$

Nat$(\cancel{\quad}, F)$

const$(\lim F)$

$\Delta(\lim F)$



$よ: \mathcal{C} \longrightarrow Fun(\mathcal{C}^{op}, Sets)$

$x \searrow \quad \Downarrow \qquad \qquad \Downarrow$

cones$(-,F) =$ Nat$(\Delta(-), F)$

$\lim F \longmapsto$ $\cong$ (?).

$\mathcal{C}(-, \lim F)$

Yoneda says that this is fully faithful... ie for any $x \in \mathcal{C}$...

$$\underset{\mathcal{C}}{Hom}(x, \lim F) \cong \underset{\underset{Fun(\mathcal{C}^{op}, Set)}{Hom}}{Nat}\big(\underset{よ_x}{\mathcal{C}(-,x)}, \underset{よ_{\lim F}}{\mathcal{C}(-,\lim F)}\big) \quad \forall x \in \mathcal{C}.$$

ex// Show that ..

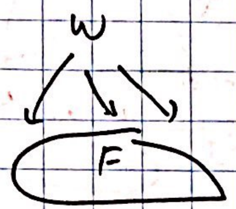
cones $\rightsquigarrow$ use u.p. limit

? $\rightsquigarrow$ cones(x,F)

$\text{cones}(-, F) \overset{?}{\cong} \mathcal{C}(-, \lim F)$

$: \mathcal{C}^{op} \to \text{Set}.$

given a cone.



$\mathcal{C}(x, \lim F)$: {set of morphisms $x \to \lim F$}

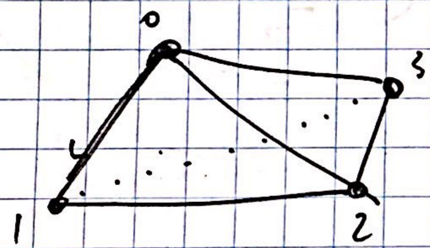
$\text{Hom}_{\mathcal{C}}(x, \lim F)$

$\text{cone}(W, F) = \{\text{set of nat'l trans. } \text{const}_W \Rightarrow F\}$.

λ is a cone

$W \xrightarrow{\exists!} \lim F$

u.p. of limit.



Can break up into ...

points $= \{0, 1, 2, 3\}$.

lines $= \{0 \to 1,\ 0 \to 2,\ 0 \to 3,\ 1 \to 2,\ 1 \to 3,\ 2 \to 3\}$.

faces $= \{\ \ldots\ \}$.

ingredients.

There are 'instructions for fitting these in to construct

(ingredients + instructions) for "simplicial set".