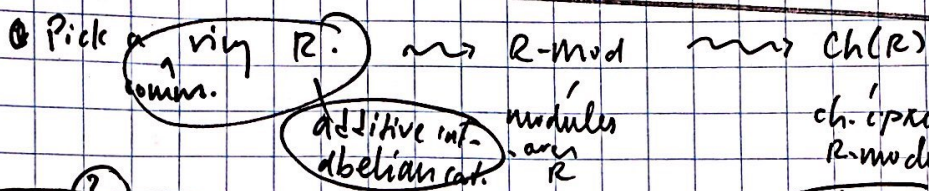


Two ples.
9/21/22.

homotopy theory \longleftrightarrow homology.

derived functors - Ext, Tor.
- what is "abelianness"?

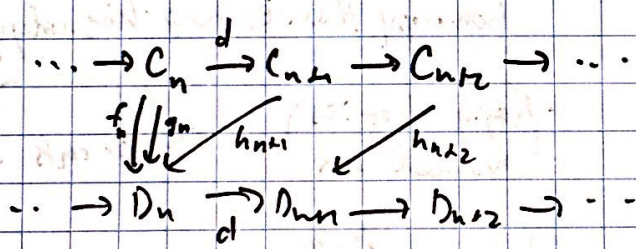
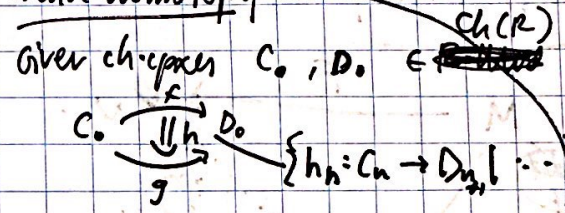
model & co-cats.
- read: Groth
Hovey.
Dugger-Spaliński



Top \leftarrow $\text{Ch}(R)$
Noori - Triangulated cats?

Q: What should be weak eq's?

(A) homology invariant - chain homotopy



define "homotopy category of ch-cpxs"

$\text{Ch}(R) \rightarrow K(R)$

$K(R)$ - ob = ch-cpxs?
mor: $\text{Hom}_{K(R)}(C_0, D_0)$

$\text{Hom}_{\text{Ch}(R)}(C_0, D_0) / \sim_{\text{ch-htpy}}$

But the notion of chain homotopy turns out not to be the one we want (in some ways it does, e.g. if $(f \sim_{\text{ch-htpy}} g) \Rightarrow \bar{f} = \bar{g}$ are the same in homology).

Turns out that the "right" notion of htpy is quasi-isomorphism

Def. ~~maps $f, g: C_0 \rightarrow D_0$~~ A map $f: C_0 \rightarrow D_0$ is a q-iso. if the induced maps $f_n: H_n(C_0) \rightarrow H_n(D_0)$ are isomorphisms (of abelian gps).

\rightsquigarrow describe a cat.

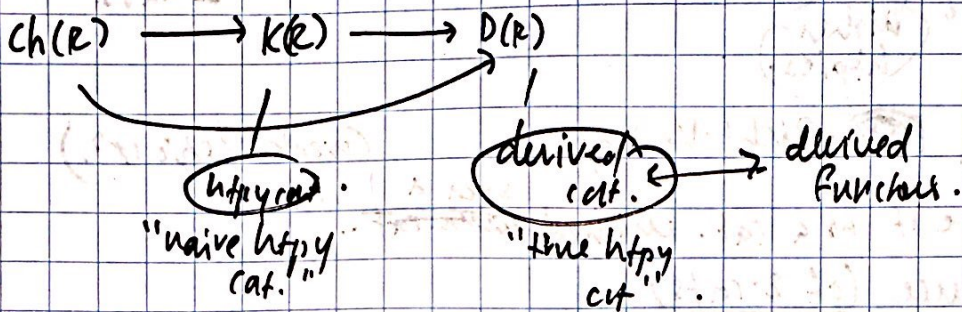
This turns out to be the "right" notion of weak eq. of ch. exes.

we can describe $D(R)$

ob = ch. exes.

$$\text{Hom}_{D(R)}(C_0, D_0) = \text{Hom}_{\text{Ch}(R)}(C_0, D_0) / \sim \text{q.iso.}$$

derived cat. of a ring (ab. cat.)



- Balchin
- Hovey
- Anilen - Homotopical Algebra

HA Ch. $D(R)$: " ∞ -derived cat."

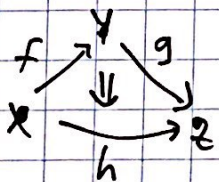
• combinatorial model cats.

(locally) presentable ∞ -categories.
~~accessibility~~
 accessible.

"Jelt Smith's theorem"
 "generated by small data"
 i.e. "easy" to get a handle on.

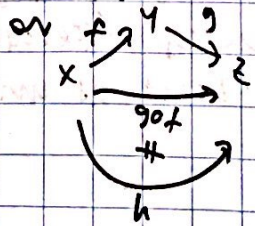
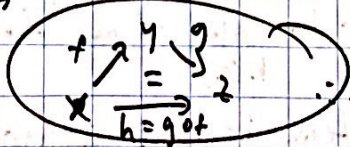
model cat. $\xrightarrow{\text{simplicial localization / Dwyer-Kan}}$ "underlying ∞ -cat."

\mathcal{C} : ∞ -cat. \rightarrow \mathcal{C}_1 : a 1-cat.
 \mathcal{C}_2 : a 2-cat.
 \vdots



$h_2 \cdot e^2$
 $h_3 \cdot e$
 Riehl-Verity "Elements of ∞ -cat. theory" ch. 1.

in \mathcal{C}_1 should this be a commutative



$\mathcal{C} : \infty\text{-cat.}$

$h_3 \mathcal{C}$

$ob = \mathcal{C}_0$

$mor = \mathcal{C}_1$

$2\text{-mor} = \mathcal{C}_2$

$3\text{-mor} = \mathcal{C}_3 / \sim$

higher
htpies.

vs.

$h_3^{en} \mathcal{C}$

(??)

describe as a

cat. enriched

over (strict? / weak?)

2-cats.

ex/1.

- look up $h_2 \mathcal{C}$ in ~~Richl~~-Verity. (weak? / strict?)
- ~~Describe~~ describe it as a cat. enriched over a 1-cat.
- show the equivalence (of 2-cats)

∞ -cats. — strict (quasi-categories)

(n -morphisms for all n)

(as simplicially enriched cats.

ie. categories whose hom-sets are ssets

(as dg-cats.

ie. cats whose are enriched in diff. graded ab-gps)

in HA no construct $\mathcal{D}(A)$

∞ -cat.

$h\mathcal{D}(A)$

1-cat.

$\mathcal{D}(A)$

To do:

- study the model cat. of complexes of ~~modules~~
- derived $\mathcal{D}(A)$ vs. homoth of an abelian cat. $\mathcal{K}(A)$
- relative categories as a model of ∞ -cat.
- quasi-categories (Kerodon)
- $h_2 \mathcal{C}$ the htpy 2-cat as an enriched cat ($h_3 \mathcal{C}$)